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TECHNICAL PREPRINT prepared for Thermionic Conversion Specialist Conference sponsored by the Institute of Electrical and Electronics Engineers
Cleveland, Ohio, October 26-28, 1964

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Abstract

The flow of electrons through a neutral scattering gas in a thermionic diode is analyzed for electron mean free paths comparable to the electrode separation. A perfect Lorentz gas with a rigid sphere scattering cross section is assumed. The corresponding Boltzmann equation is transformed to a pure integral equation, and self-consistent-field solutions of this equation with Poisson's equation have been computed. Density and potential variations together with saturation portions of current voltage curves are presented.

Author

Introduction

The analysis of transport effects in a thermionic diode is complicated by the strong influence on the entire system of phenomena occurring in the immediate vicinity of the electrodes. For large spacings this is seen in the artificial separation of the interelectrode space into collisionless sheaths and a collision-dominated plasma. For close spacing collisions are frequently ignored altogether. The method employed in the present analysis can be used to calculate transport effects over distances of a few mean free paths with results as accurate as the knowledge of the collision cross sections involved.

The use of integral equation formulations in the solution of Boltzmann's equation is not new. It has been suggested by Grad[1,2] and used by Willis[3] to solve the Krook model of the linearized Boltzmann equation in shear flow. By introducing a variable transformation to deal with the potential field, Willis' method has been adapted in this work to treat the flow of electrons. Solutions have been obtained for the flow from a hot emitter to a cold collector and include the effects of space charge and electron-neutral atom collisions.

General Analysis

For electrode separations that are not too large, the loss of energy by electrons due to elastic collisions with atoms is negligible. In addition, all inelastic collisions are also neglected. The Boltzmann's equation for the one-dimensional flow of a perfect Lorentz gas is written[4,5]

$$\cos\theta\left(\bar{v}\frac{\partial\bar{f}}{\partial\bar{x}} + \frac{e}{m}\frac{dV}{d\bar{x}}\frac{\partial\bar{f}}{\partial\bar{x}}\right) + \frac{e}{m}\frac{dV}{dx}\frac{\sin^2\theta}{\bar{v}}\frac{\partial\bar{f}}{\partial\cos\theta} = N\bar{v}\int\left[\bar{f}(\bar{x},\bar{v},\theta') - \bar{f}(\bar{x},\bar{v},\theta)\right]\sigma\, d\Omega' \tag{1}$$

where $\bar{x}$, $\bar{v}$, and θ represent, respectively; distance, velocity, and polar angle in velocity space; $\bar{f}$ is the electron distribution function; V the potential; N the atom density; and σ the differential cross section. The bars are used to distinguish dimensional quantities.

The density and current, respectively, are given by

$$\bar{n} = 2\pi\int_0^\infty\int_0^\pi \bar{f}\sin\theta\, d\theta\bar{v}^2\, d\bar{v} \tag{2}$$

$$\bar{j} = 2\pi\int_0^\infty\int_0^\pi \bar{f}\cos\theta\sin\theta\, d\theta\bar{v}^3\, d\bar{v} \tag{3}$$

Equation (1) with (2) is to be solved simultaneously with Poisson's equation

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$$\frac{d^2V}{d\bar{x}^2} = 4\pi e\bar{n} \tag{4}$$

Boundary conditions are

$$\left.\begin{array}{lll} \bar{x} = 0 & \cos\theta > 0 & \bar{f} = 2\bar{n}_o\left(\frac{m}{2\pi kT_o}\right)^{3/2} e^{-m\bar{v}^2/2kT_o} \\ \bar{x} = L & \cos\theta < 0 & \bar{f} = 0 \\ V(0) = 0 & V'(0) = \text{constant} & \end{array}\right\} \tag{5}$$

where $\bar{n}_o$ and T_o are the density and temperature of the emitted electrons and L is the electrode separation. The conditions on $\bar{f}$ are obtained by assuming that there is no reflection of electrons at either electrode.

On assuming isotropic scattering and introducing dimensionless variables, Eqs. (1) to (5) become

$$\mu\left(v\frac{\partial f}{\partial x} + \frac{1}{2}\frac{d\varphi}{dx}\frac{\partial f}{\partial v}\right) + \frac{1}{2}\frac{d\varphi}{dx}\frac{1-\mu^2}{v}\frac{\partial f}{\partial \mu} = -Kvf + \frac{1}{2}Kv\int_{-1}^{1} f'\, d\mu' \tag{6}$$

$$n = 2\pi\int_0^\infty\int_{-1}^{1} f\, d\mu v^2\, dv \tag{7}$$

$$J = 2\pi^{3/2}\int_0^\infty\int_{-1}^{1} f\mu\, d\mu v^3\, dv \tag{8}$$

$$\frac{d^2\varphi}{dx^2} = Cn \tag{9}$$

$$\left.\begin{array}{lll} x = 0 & \mu > 0 & f = \frac{2}{\pi^{3/2}} e^{-v^2} \\ x = 1 & \mu < 0 & f = 0 \\ \varphi(0) = 0 & \varphi'(0) = \text{constant} & \end{array}\right\} \tag{10}$$

where

$$x = \frac{\bar{x}}{L} \qquad v = \bar{v}\sqrt{\frac{m}{2kT_o}} \qquad \mu = \cos\theta \qquad \varphi = \frac{eV}{kT_o}$$

$$f = \frac{\bar{f}}{\bar{n}_o}\left(\frac{2kT_o}{m}\right)^{3/2} \qquad n = \frac{\bar{n}}{\bar{n}_o} \qquad J = \frac{\bar{J}}{\bar{n}_o}\sqrt{\frac{\pi m}{2kT_o}} \equiv \frac{\bar{J}}{\bar{J}_o}$$

$$K = \frac{L}{\lambda} \qquad C = 4\pi e^2\,\frac{\bar{n}_o L^2}{kT_o}$$

and the mean free path λ is given by

$$\lambda = (4\pi N\sigma)^{-1} \tag{11}$$

In order to transform Eq. (6) to a pure integral equation, transform variables from (x,v,μ) to (x,w,u), where

$$\left.\begin{aligned} w &= v^2 - \varphi(x) \\ u &= \mu^2 v^2 - \varphi(x) \end{aligned}\right\} \tag{12}$$

In addition, denote f by $f^{\pm}$ for $\mu \gtrless 0$, and define

$$f^{\pm} = \frac{2}{\pi^{3/2}} e^{-w} \psi^{\pm} \tag{13}$$

Then Eqs. (6) to (8) become

$$\frac{\partial \psi^{\pm}}{\partial x} = \mp K\alpha(x)\psi^{\pm} \pm \frac{1}{2} \frac{K}{\sqrt{u + \varphi(x)}} S(x,w) \tag{14}$$

$$n = \frac{2}{\sqrt{\pi}} \int_{-\varphi(x)}^{\infty} e^{-w} S(x,w) dw \tag{15}$$

$$J = \int_{-\varphi(x)}^{\infty} \int_{-\varphi(x)}^{w} e^{-w}(\psi^+ - \psi^-) du\, dw \tag{16}$$

with

$$\alpha(x) = \sqrt{\frac{w + \varphi(x)}{u + \varphi(x)}} \tag{17}$$

$$S(x,w) = \frac{1}{2} \int_{-\varphi(x)}^{w} (\psi^+ + \psi^-) \frac{du}{\sqrt{u + \varphi(x)}} \tag{18}$$

The boundary conditions of Eq. (10) become

$$\left.\begin{aligned} x = 0 \qquad & \psi^+ = 1 \\ x = 1 \qquad & \psi^- = 0 \\ \varphi(0) = 0 \qquad & \varphi'(0) = \text{constant} \end{aligned}\right\} \tag{19}$$

Equation (14) is formally integrated to give

$$\psi^{\pm} = A^{\pm}(w,u) \exp\left[\pm \int_{x_0^{\pm}}^{x} K\alpha(x')dx'\right] \pm \frac{1}{2} \int_{x_0^{\pm}}^{x} \frac{K}{\sqrt{u + \varphi(x)'}} S(x',w) \exp\left[\pm \int_{x'}^{x} K\alpha(x'')dx''\right] dx' \tag{20}$$

In general, K is a function of $w + \varphi(x)$. The quantities $A^{\pm}(w,u)$ and $x_0^{\pm}$ are to be determined from the boundary conditions and such other conditions as particular potential configurations may impose on the distribution function.

In order to solve for $S(x,w)$, Eq. (20) is substituted into Eq. (18). After rearranging, a linear integral equation is obtained in $S(x,w)$. In many respects the theory is similar to that of radiative transfer.[6] These equations are now applied to a particular case.

Rigid Sphere Scattering With a Monotonic Increasing Potential

For this case K is a constant and $\varphi(x) \geq 0$. The complete conditions on ψ^+ are for $u \geq 0$,

$$\left.\begin{array}{ll} x = 0 & \psi^+ = 1 \\ x = 1 & \psi^- = 0 \end{array}\right\} \tag{21}$$

for $u < 0$,

$$\left.\begin{array}{ll} x = x_t & \psi^+ = \psi^- \\ x = 1 & \psi^- = 0 \\ \multicolumn{2}{c}{u + \varphi(x_t) = 0} \end{array}\right\} \tag{22}$$

The point x_t is the turning point for electrons of "energy" u in the electric field.

Applying conditions (21) and (22) to Eq. (20) gives

for $u \geq 0$,

$$\left.\begin{array}{l} \psi^+ = \exp\left[-K \int_0^x \alpha(x')dx'\right] + \frac{1}{2} K \int_0^x \frac{S(x',w)}{\sqrt{u + \varphi(x')}} \exp\left[-K \int_{x'}^x \alpha(x'')dx''\right] dx' \\ \psi^- = \frac{1}{2} K \int_x^1 \frac{S(x',w)}{\sqrt{u + \varphi(x')}} \exp\left[-K \int_x^{x'} \alpha(x'')dx''\right] dx' \end{array}\right\} \tag{23}$$

for $u < 0$,

$$\left.\begin{array}{l} \psi^+ = \frac{1}{2} K \int_{x_t}^1 \frac{S(x',w)}{\sqrt{u + \varphi(x')}} \exp\left[-K \int_{x_t}^{x'} \alpha(x'')dx'' - K \int_{x_t}^{x} \alpha(x'')dx''\right] dx' \\ \qquad + \frac{1}{2} K \int_{x_t}^x \frac{S(x',w)}{\sqrt{u + \varphi(x')}} \exp\left[-K \int_{x'}^x \alpha(x'')dx''\right] dx' \\ \psi^- = \frac{1}{2} K \int_x^1 \frac{S(x',w)}{\sqrt{u + \varphi(x')}} \exp\left[-K \int_x^{x'} \alpha(x'')dx''\right] dx' \end{array}\right\} \tag{24}$$

After substitution of Eqs. (23) and (24) into Eq. (18) and interchanging the order of the x and u integrations, the following integral equation for $S(x,w)$ is obtained:

$$S(x,w) = F(x,w) + \frac{1}{2} K \int_0^1 H(x,x',w)S(x',w)dx' \tag{25}$$

where

$$F(x,w) = \frac{1}{2} \int_0^w \exp\left[-K \int_0^x \alpha(x')dx'\right] \frac{du}{\sqrt{u + \varphi(x)}} \tag{26}$$

$$H(x,x',w) = \frac{1}{2}\int_b^w \exp\left[-K\left|\int_{x'}^{x} \alpha(x'')dx''\right|\right] \frac{du}{\sqrt{u + \varphi(x)}\ \sqrt{u + \varphi(x')}}$$

$$+ \frac{1}{2}\int_b^0 \exp\left[-K\int_{x_t}^{x} \alpha(x'')dx'' - K\int_{x_t}^{x'} \alpha(x'')dx''\right] \frac{du}{\sqrt{u + \varphi(x)}\ \sqrt{u + \varphi(x')}} \tag{27}$$

and

$$b = \begin{cases} -\varphi(x') & x' < x \\ -\varphi(x) & x' > x \end{cases} \tag{28}$$

Equation (25) can be solved by iteration in the form

$$\left.\begin{aligned} S^{(o)}(x,w) &= F(x,w) \\ S^{(n)}(x,w) &= F(x,w) + \frac{1}{2} K \int_0^1 H(x,x',w)S^{(n-1)}(x',w)dx' \end{aligned}\right\} \tag{29}$$

Once $S(x,w)$ has been found, the density is given by Eq. (15) as

$$n(x) = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-w}S(x,w)dw \tag{30}$$

and the current, evaluated at the emitter, by Eqs. (16), (23), (24), and (26) as

$$J = 1 - K\int_0^\infty e^{-w}\int_0^1 S(x',w)F(x',w)dx'\ dw \tag{31}$$

Note that $w \geq 0$ as each particle's total energy is conserved.

Results

Equation (25) with (30) has been solved self-consistently with Eq. (9) on the IBM 7094 computer. The equations were solved for $L/\lambda = 0, 0.5, 1, 2,$ and 5 and C [Eq. (9)] = 50. An emitter temperature of 1900° K, a spacing of 10μ, and an emission of 10 amp/cm^2 would give a C of about 50. Typical density and potential curves are shown in Figs. 1 and 2, while current voltage curves are presented in Fig. 3.

Examination of the inhomogeneous function $F(x,w)$ in Eq. (25) shows that it has an infinite slope at $x = 0$. This shows up in the steep slopes of the density curves (Fig. 1) at $x = 0$. More accurate computation would show that these slopes are actually infinite.

The results shown in Fig. 3 agree quite closely with an independent Monte Carlo calculation (presented in the paper by C. M. Goldstein).

Conclusions

The method of solution of the Boltzmann equation presented in this paper has been shown to be an accurate and quite feasible means of analyzing transport phenomena over distances of a few mean free paths. The same method can be used to handle ion transport phenomena and, by modifying the boundary conditions, to treat the transition region between an electrode and a plasma.

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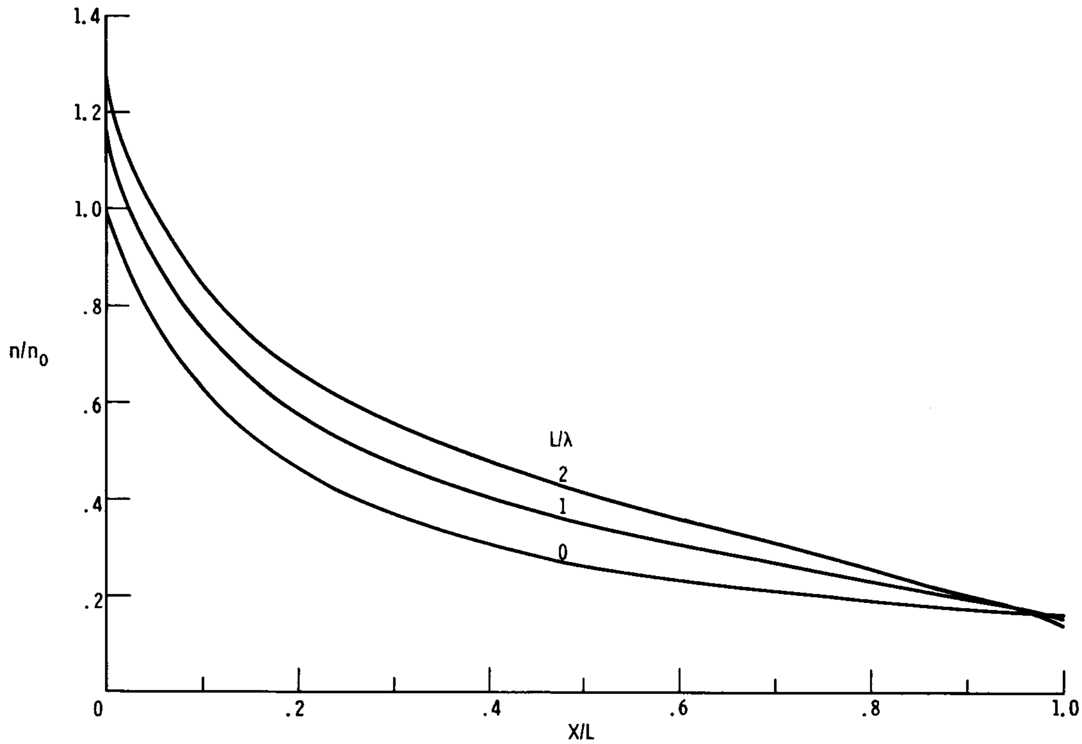

Fig. 1. - Density as function of distance. C = 50; $\varphi'(0) = 0$.

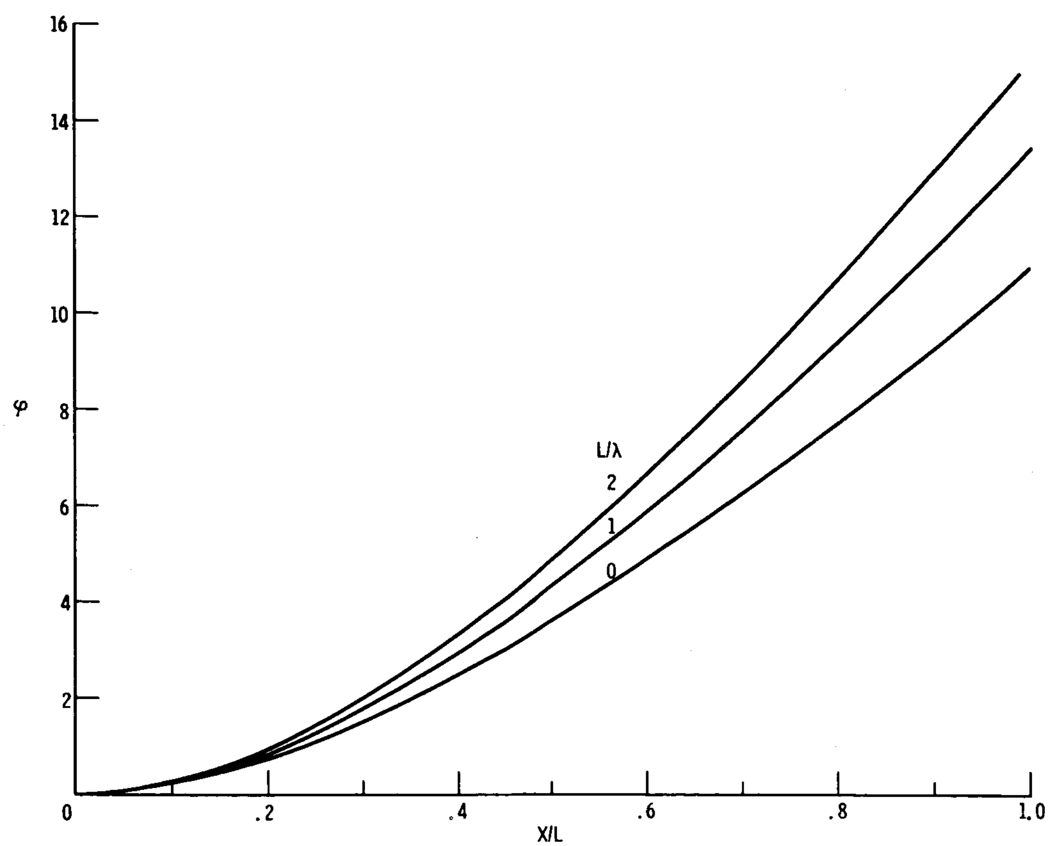

Fig. 2. - Potential as function of distance. C = 50; $\varphi'(0) = 0$.

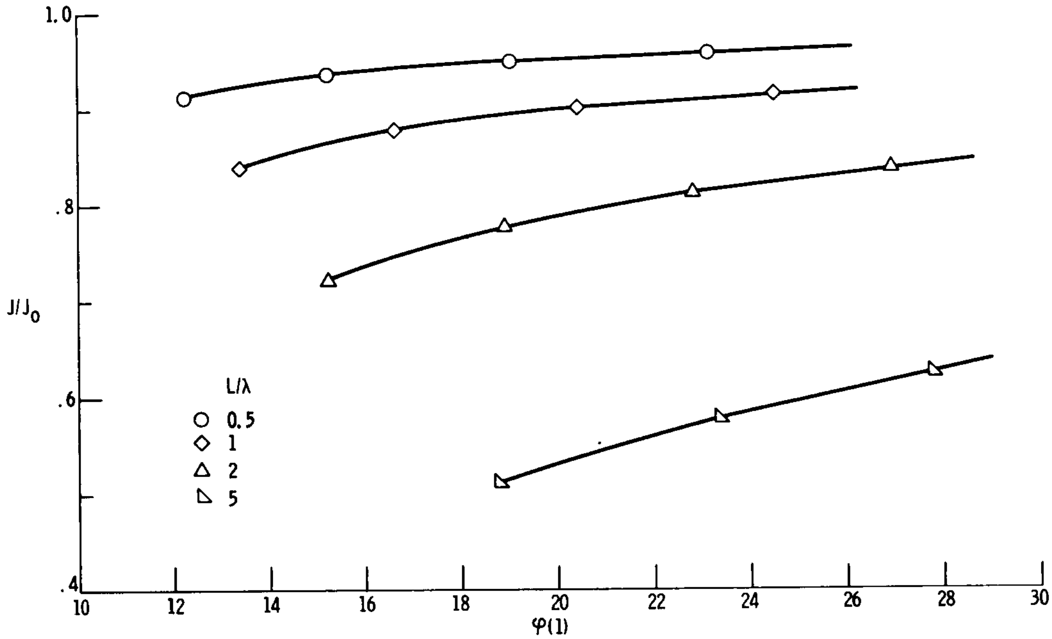

Fig. 3. - Current as function of anode voltage. C = 50.